Hydrodynamics of the Lieb-Liniger Gas in One Dimension

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ABSTRACT

The study of many-particle systems is usually very hard; different types of approximations are often needed to make any progress. Especially hard are systems with non-linear behavior that are sensitive to initial conditions. However, some non-linear systems have astonishingly periodic motion despite the complex nature of their interactions. We focus on the paradigmatic system of cold bosons in one dimension called the Lieb-Liniger model, a highly tunable system that can provide insight on a variety of condensed matter systems. In addition, the Quantum Newton's Cradle experiment by Kinoshita, Wenger, and Weiss at Penn State recently showed that these systems are actually realizable in the practical world. We detail a molecular dynamics simulator that is able to capture the hydrodynamic description of the gas using classical physics and report our success in implementation.

Lieb-Liniger Model

The Lieb-Liniger Model (LL) describes a system of N bosons that interact via contact with strength c.

$$-\sum_{i=1}^{N}\left(\frac{\partial^2}{\partial x_i^2}+2c\sum_{i<j}^{N}\delta(x_i-x_j)\right)\Psi=E\Psi$$

Defined in 1D with periodic boundary conditions (i.e. on a ring)

Quantum Newton's Cradle

The QNC experiment [1] describes a LL gas clustered at the bottom of a potential well. Half of the bosons are kicked left (and the other half to the right) and the gas is evolved over time.

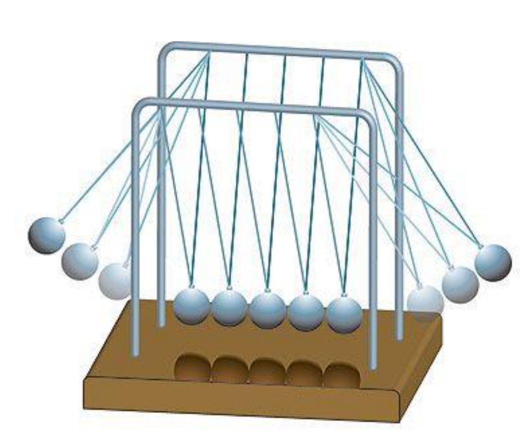

FIG 1 (a): The Newton's Cradle is a physics based toy that demonstrates conservation of energy and momentum. It is a classical system of many particles that has regular, periodic motion.

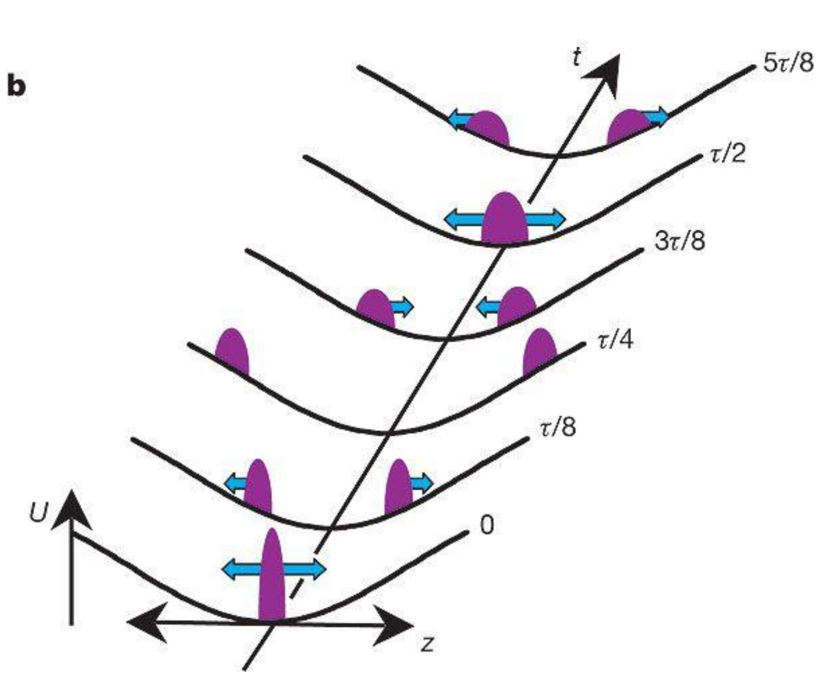

FIG 1 (b): Distribution of bosons over time in The Quantum Newton's Cradle setup. The purple area shows the bosons' positions and the blue arrows indicate their velocities. The bosons oscillate in an anharmonic potential well.

Image Taken from Kinoshita et. al. [1]

The QNC experiment is interesting because the bosons have regular periodic motion (as opposed to thermalization and chaos). The integrability (infinite amount of conserved quantities) of the LL model is the cause of this type of motion and the experiment showed that these idealized systems are possible to realize in the practical world.

Flea Gas Algorithm

Due to integrability, the QNC system can be modeled using generalized hydrodynamics (GHD). GHD groups particles into fluid cells that contain interactions slowing the gas down to an effective velocity [2]

$$v^{\mathrm{eff}}(v)=v+\int\left(d(v,w)\rho(w)[v^{\mathrm{eff}}(v)-v^{\mathrm{eff}}(w)]\right)dw$$

This velocity reduction can be, instead, thought of as instantaneous, backward "quantum jumps" that occur at particle collisions whose distances are dependent on the colliding velocities

$$d(v,w)=\frac{-2c}{(v-w)^2+c^2}$$

Jumps can cause more collisions which can cause a chain reaction of jumps that are likened to fleas.

RESULTS

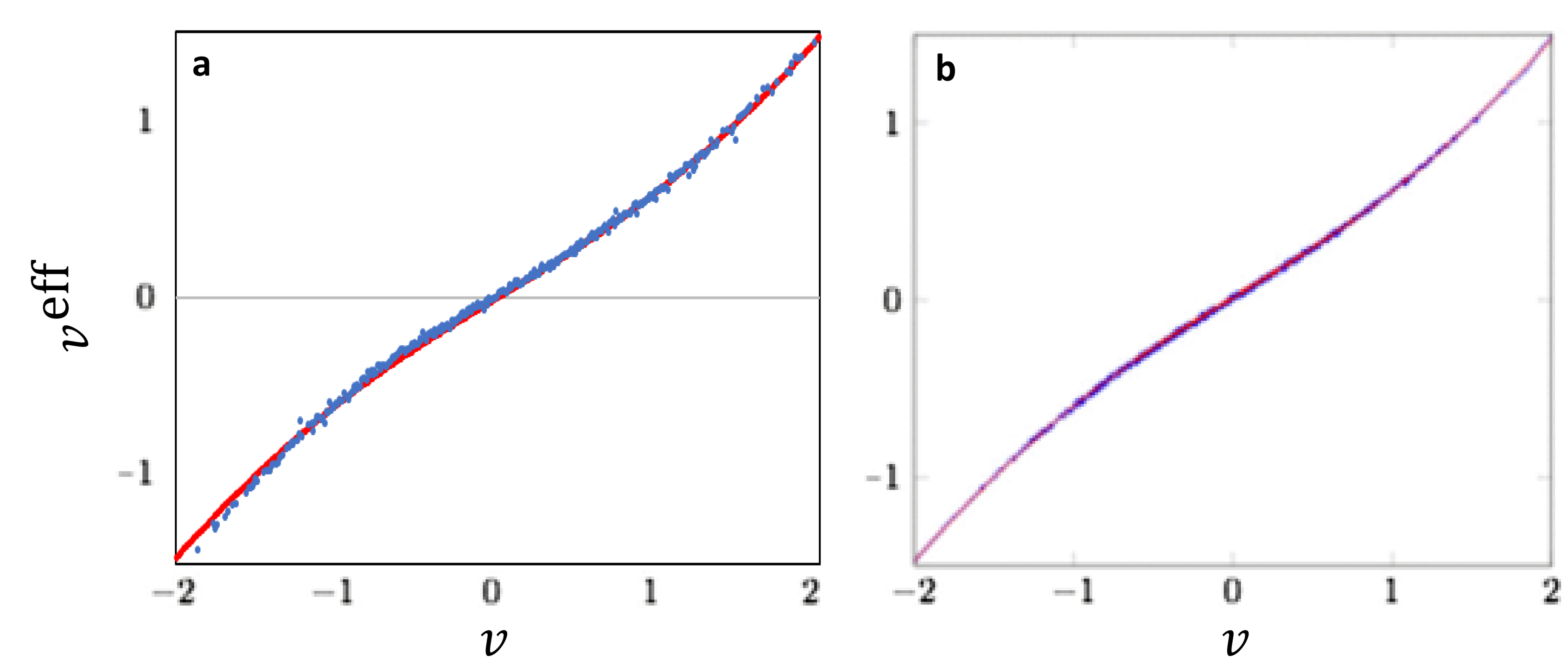

FIG 2: The effective velocity of a LL gas of 1,500 particles is evaluated using a truncated gaussian initial distribution of velocities and a uniformly spaced gas under periodic boundary conditions. The red curve is the exact curve given by the effective velocity equation and the blue points are evaluated using the flea gas algorithm. (a) Our results seem to be in decent agreement with (b) the results of the paper [2] where the flea gas algorithm is outlined.

FUTURE WORK

Equipped with an algorithm to compute the dynamics of the LL gas out of equilibrium, we can now study the gas within the QNC setup. In particular, we hope to explore a system of two LL gases that interact with each other in such a way that breaks integrability. This is important because the dipolar QNC (recently created by Tang et. al. [3] consisting of highly magnetic dysprosium atoms) is an experimental realization of exactly the type of interacting system we wish to study.

REFERENCES

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