Using the variational quantum eigensolver to study a lattice fermion model

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ABSTRACT

Although computing power has been steadily rising over the years, there are some problems that classical computers simply cannot solve. Many lattice gauge theories used in nuclear physics contain such computationally complex problems that cannot be solved in a reasonable time. However, advances in low-temperature physics allow these models to be simulated by ultracold atoms. These cold atom systems can be studied by quantum computers which have recently been made available to the public via the cloud from IBM. Since quantum computers have the capability to solve certain problems exponentially faster than classical computers, we decided to study a lattice fermion model using the IBM's Qiskit. Specifically, we used the variational quantum eigensolver algorithm to find the ground state energy of several similar systems. We report our findings using both a local simulator (Qiskit) and an actual quantum computer (IBMQ).

QUANTUM COMPUTING

Quantum computers are machines that rely on the physics of quantum mechanical two-level systems to perform computations. They operate in a fundamentally different way than classical computers by using qubits instead of bits.

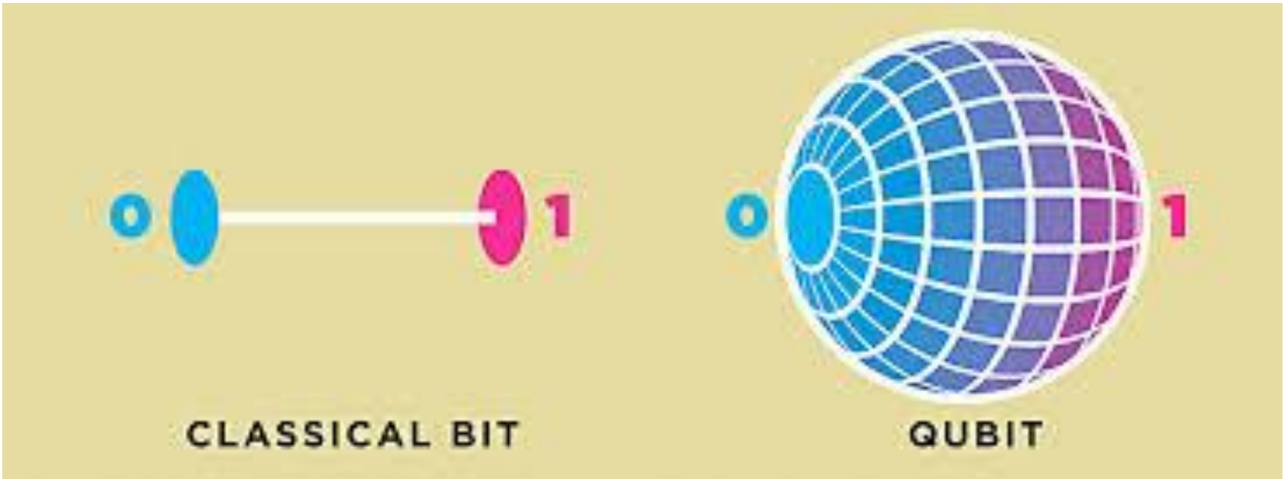

Fig 1: Qubits exhibit superposition and entanglement which quantum computers leverage to perform computations that have the capability to outclass modern supercomputers.

Image Credit: https://www.austinchronicle.com/screens/2019-04-19/quantum-computing-101-a-beginners-guide-to-the-mind-bending-new-technology/

VARIATIONAL QUANTUM EIGENSOLVER

We use IBM's quantum computing platform Qiskit [1] to run the variational quantum eigensolver (VQE) algorithm [2] on a lattice model in nuclear physics. The VQE finds the lowest eigenvalue (ground state energy) of a Hermitian matrix (hamiltonian). For each run, we specify the number of shots (repeated quantum calculation to generate adequate statistics) and trials (optimization steps).

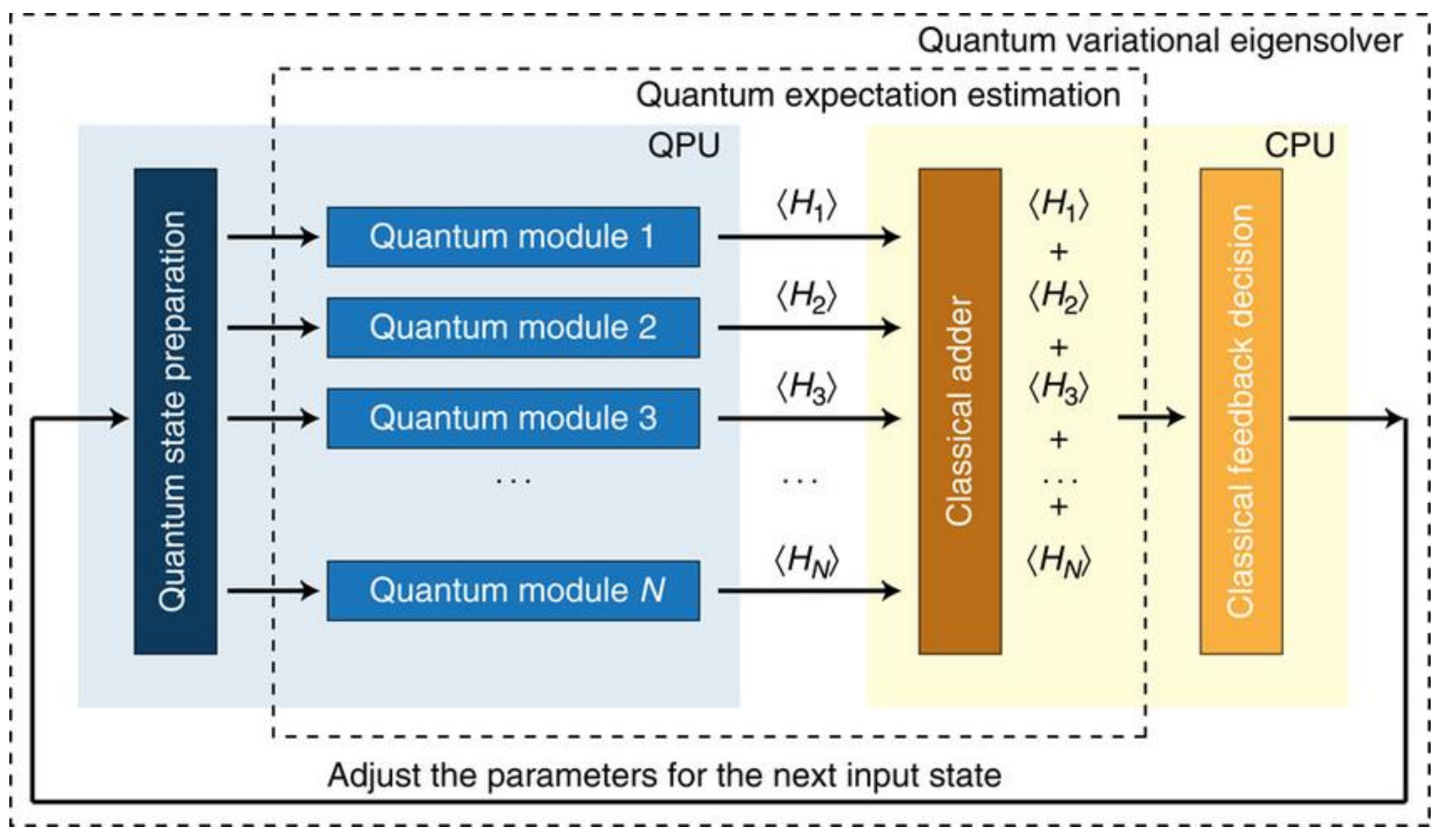

Fig 2: The flow chart of the VQE algorithm (from Peruzzo et al.) which is based on the quantum mechanical variational method. The VQE takes advantage of quantum hardware that can perform parallel computation and is guaranteed to find an upper bound for the ground state energy.

MODEL

Our model describes four fermions in a square lattice where nearest neighbors can interact through a boson link. The hamiltonian is given by

$$\hat{H} = \left(\bigotimes^{j-1} 1_3\right) \otimes \frac{\hat{P}^2}{2} \otimes \left(\bigotimes^{n_B-j} 1_3\right) \otimes 1_{16} - \sum_{i,j}^{4} c^\dagger_{Fi} A_{i,j} c_{Fj}$$

Here, $\otimes$ is the tensor product, $\hat{P}$ is the position basis representation of momentum, $A_{i,j}$ is the adjacency matrix accounting for fermion-boson interactions, and c_{Fj} is the fermion operator.

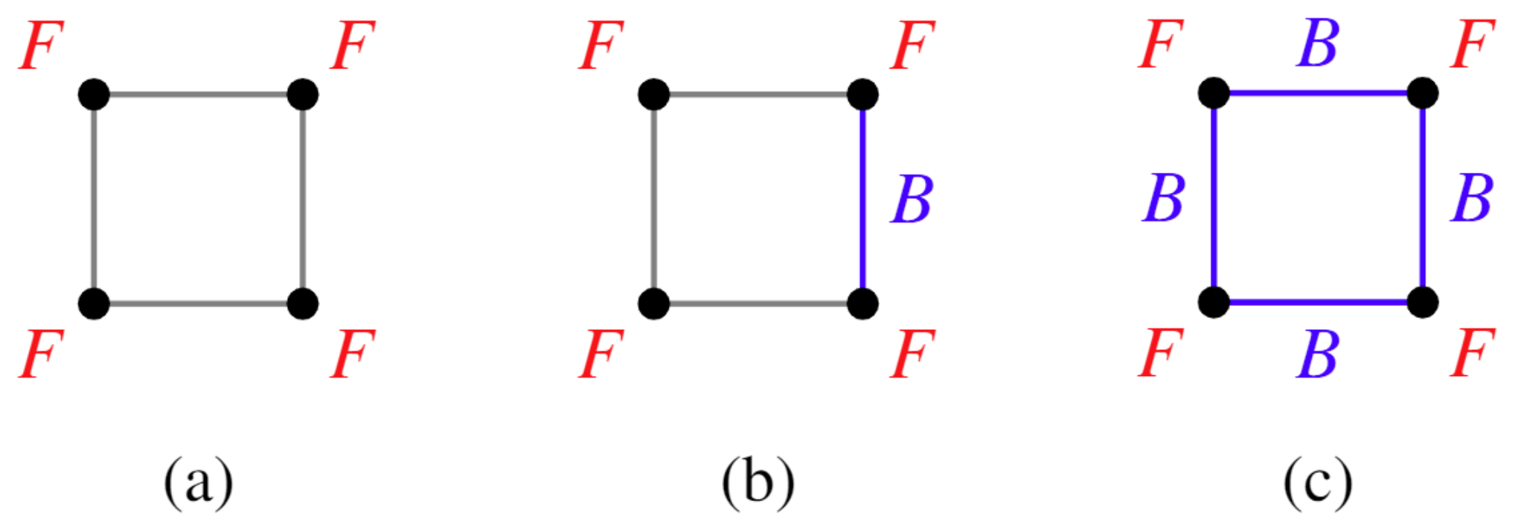

Fig 3: The four fermion lattice with (a) no bosons, (b) one boson, and (c) four identical bosons.

RESULTS

Using the local simulator, we report the VQE converge plots which show the estimate of the ground state energy (E_0) as the optimization subroutine progresses.

E_0 for 4 fermions with 0 bosons

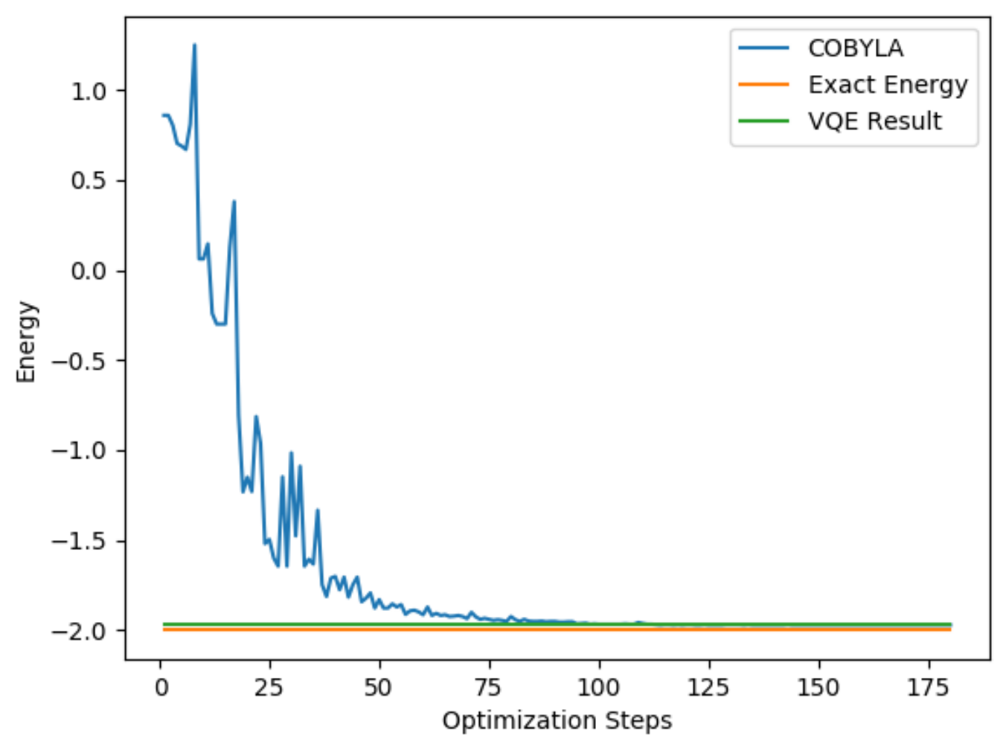

Fig 4: VQE convergence plot for Fig 3(a) on a local simulator (500 shots / 500 trials)

E_0 for 4 fermions with 1 boson

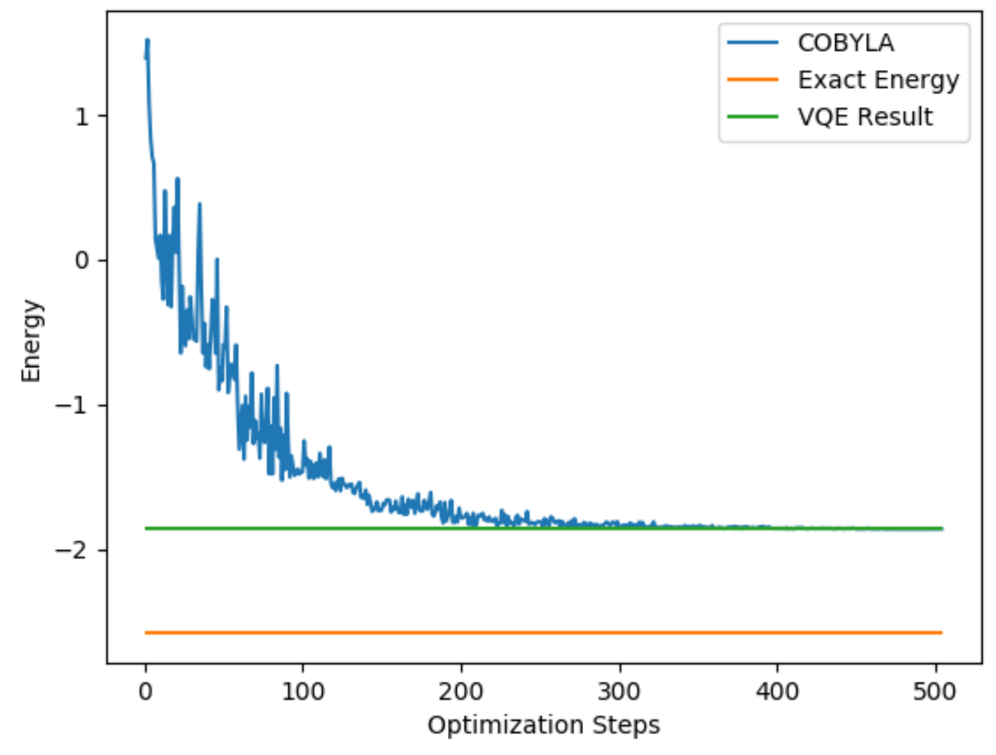

Fig 5: VQE convergence plot for Fig 3(b) on a local simulator (5,000 shots / 5,000 trials)

Fig 6 shows that IBMQ cannot run through many optimization steps and is not able to converge to correct value. This is most likely due to limitations on what the public can access. The quantum computer ran at a rate around 60x slower than the local simulator for obtaining ground state energy of Fig 3(a).

E_0 for 4 fermions with 0 bosons (IBMQ)

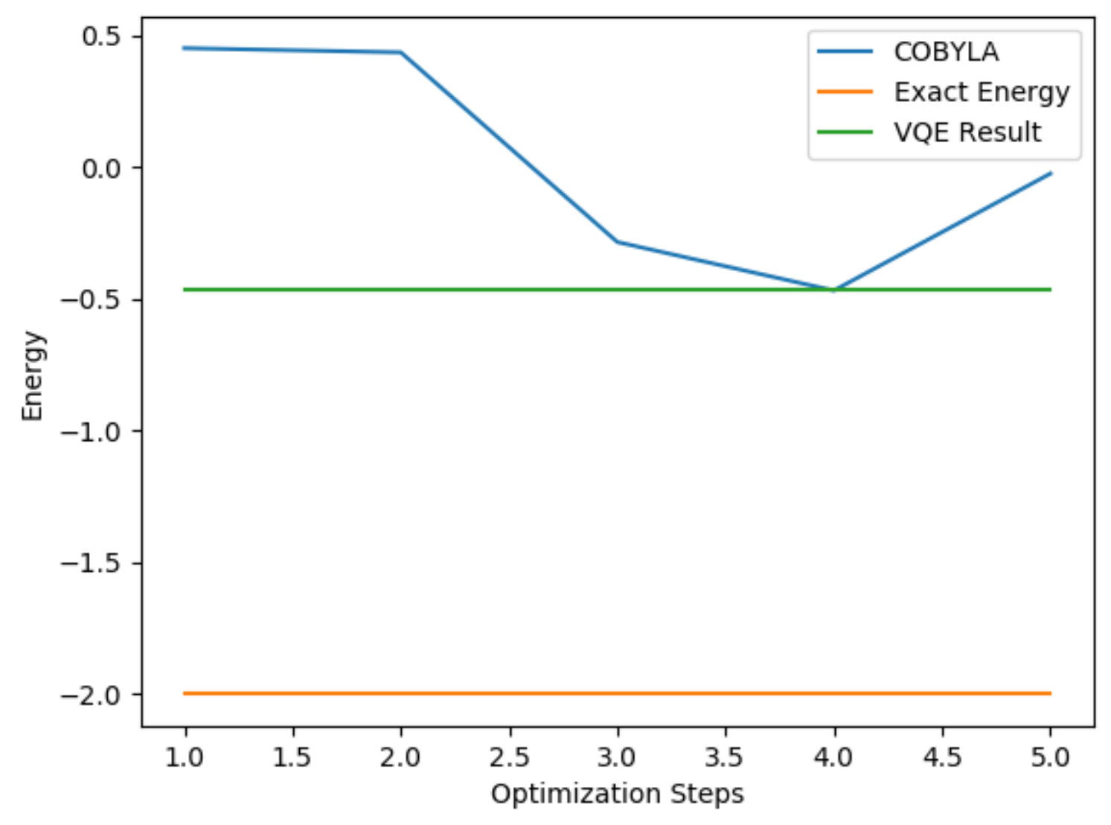

Fig 6: VQE convergence plot for Fig 3(a) on IBM quantum computer (500 shots / 5 trials)

FUTURE WORK

We hope that we can find better results for more complicated systems in the future by finding a good trial wave function for the VQE and finding a way to simulate many qubits without creating large matrices.

REFERENCES

[1] A. Peruzzo et al., "A variational eigenvalue solver on a photonic quantum processor," Nature Communications 5, 4213 (2014).
[2] G. Aleksandrowicz et al., "Qiskit: An Open-source Framework for Quantum Computing," (2019).

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